What Makes a Scientific Explanation Distinctively Mathematical?

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ABSTRACT

Certain scientific explanations of physical facts have recently been characterized as distinctively mathematical–that is, as mathematical in a different way from ordinary explanations that employ mathematics. This article identifies what it is that makes some scientific explanations distinctively mathematical and how such explanations work. These explanations are non-causal, but this does not mean that they fail to cite the explanandum's causes, that they abstract away from detailed causal histories, or that they cite no natural laws. Rather, in these explanations, the facts doing the explaining are modally stronger than ordinary causal laws (since they concern the framework of any possible causal relation) or are understood in the why question's context to be constitutive of the physical arrangement at issue. A distinctively mathematical explanation works by showing the explanandum to be more necessary (given the physical arrangement in question) than ordinary causal laws could render it. Distinctively mathematical explanations thus supply a kind of understanding that causal explanations cannot.

1 Introduction

Mathematics figures in many scientific explanations. But several philosophers have recently characterized certain explanations of physical facts as

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distinctively mathematical–that is, as mathematical in a different way from ordinary scientific explanations employing mathematics. My goal in this article is to understand what makes a scientific explanation distinctively mathematical and how these explanations work.

These explanations target physical facts rather than mathematical theorems. Mathematical explanations in mathematics (such as arguments that not only prove some mathematical theorem, but also explain why it holds) form another topic (see Mancosu [2008]). (However, as I discuss in Section 5, Steiner's ([1978]) account of distinctively mathematical explanations in science appeals to the notion of a mathematical explanation in mathematics.)

In trying to characterize 'distinctively mathematical' explanations in science, I am not trying to explicate the meaning of the term 'distinctively mathematical' so as to agree with intuitions about the term's proper application. We may well have no pretheoretic notions at all regarding what a 'distinctively mathematical' scientific explanation would be. Furthermore, my aim is not to explicate the meaning of 'distinctively mathematical' so as to fit this term's use in scientific practice. This term is not commonly used in science. Nevertheless, the task of characterizing 'distinctively mathematical' scientific explanations aims to fit both certain intuitions and certain features of scientific practice.

In Section 2, I will present several examples of scientific explanations that are mathematical in a way that intuitively differs profoundly from ordinary scientific explanations employing mathematics. One goal of this article is to explore this apparent difference in order to see whether it withstands careful scrutiny. I will use the term 'distinctively mathematical' to mark this apparent difference. Ultimately, I will suggest that there is in fact a fundamental difference between these explanations. I will offer an account of what makes certain scientific explanations but not others 'distinctively mathematical' that aims to accord with our intuitions about which scientific explanations are alike and which fundamentally differ.

An account of 'distinctively mathematical' explanations also aims to fit scientific practice by deeming to be explanatory only hypotheses that would (if true) constitute genuine scientific explanations. More ambitiously, an account aims to specify how 'distinctively mathematical' explanations work–that is, to identify the source of their explanatory power.

After the D-N model (Hempel and Oppenheim [1948]) was shown to count various non-explanations as explanatory (in connection with such notorious counterexamples as the flagpole, the eclipse, and the barometer—see Salmon [1989], pp. 46–50), philosophers frequently suggested that to avoid many of these problems for the D-N model, we should recognize the central role that causal relations play in scientific explanations. Indeed, many philosophers went further by suggesting that all scientific explanations (or, at least, all

explanations of particular events or singular facts) are 'causal explanations'. For example, Wesley Salmon has written:

> To give scientific explanations is to show how events and statistical regularities fit into the causal structure of the world. (Salmon [1977], p. 162)

> Causal processes, causal interactions, and causal laws provide the mechanisms by which the world works; to understand why certain things happen, we need to see how they are produced by these mechanisms. (Salmon [1984], p. 132)

The same note has been sounded by many other philosophers:

> Here is my main thesis: *to explain an event is to provide some information about its causal history.* (Lewis [1986], p. 217; endorsed, for instance, by Jackson and Pettit [1992], pp. 12–3)[1]

> Causal explanation is the unique mode of explanation in physics. (Elster [1983], p. 18)

> The explanation of an event describes the 'causal structure' in which it is embedded. (Sober [1984], p. 96)

Recent accounts of scientific explanation (such as Woodward [2003] and Strevens [2008]) have continued to emphasize that scientific explanations work by describing causal connections.

I will argue that this view of scientific explanation cannot do justice to 'distinctively mathematical' explanations; they are non-causal scientific explanations. In so arguing, I am not appealing to some account of what makes an explanation 'causal' that aims to fit either some pretheoretic intuitions about which explanations are 'causal' or some scientific practice of labeling certain explanations 'causal'. Rather, I am elaborating the notion of 'causal' explanation that is employed by those philosophers whom I have quoted as contending that all scientific explanations are causal. Distinctively mathematical explanations are 'non-causal' because they do not work by supplying information about a given event's causal history or, more broadly, about the world's network of causal relations. Such an explanation works instead by (roughly) showing how the fact to be explained was inevitable to a stronger degree than could result from the causal powers bestowed by the possession of various properties. If a fact has a distinctively mathematical explanation, then the modal strength of the connection between causes and effects is insufficient to account for that fact's inevitability.

Thus, the importance of understanding how distinctively mathematical explanations work does not derive from the significance of any intuitions we may have regarding what makes an explanation 'distinctively

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[1] Though Lewis confines his remark to the explanation of a particular event, he also says that to explain a general kind of event (e.g. why struck matches light) is to describe similarities in the causal histories of the events of that kind (Lewis [1986], p. 225).

mathematical' or 'causal'. Rather, its importance lies in what it reveals about the kinds of scientific explanations there are and the limits of philosophical accounts that place causal relations at the center of all scientific explanations.

In Section 2, I give some examples of 'distinctively mathematical' explanations that suggest how different these explanations are from ordinary scientific explanations that use mathematics. I then sketch what I believe their fundamental difference to be. In Section 3, I consider whether distinctively mathematical explanations are set apart by their failure to cite causes. I argue that, on the contrary, many ordinary scientific explanations fail to give the explanandum's causes and at least some distinctively mathematical explanations do cite the explanandum's causes. Having adopted a broad notion of what makes an explanation 'causal', I argue in Section 4 that distinctively mathematical explanations in science are non-causal. I also argue that we must narrow the explananda very carefully before we can agree with those philosophers who have characterized certain natural-selectionist explanations as distinctively mathematical. In Section 5, I present my account of how distinctively mathematical explanations work. I argue that even when such an explanation appeals to a contingent law of nature, it works by showing the explanandum to be necessary in a stronger sense than any causal explanation could. I conclude in Section 6.

2 Some Distinctively Mathematical Scientific Explanations

The best way to approach our topic is to give several examples of scientific explanations that appear to be distinctively mathematical. Here is a very simple example (inspired by Braine [1972], p. 144):

> The fact that twenty-three cannot be divided evenly by three explains why it is that Mother fails every time she tries to distribute exactly twenty-three strawberries evenly among her three children without cutting any (strawberries!).

The explanation seems no less distinctively mathematical when certain contingent facts are also included in the explanans:

> That Mother has three children and twenty-three strawberries, and that twenty-three cannot be divided evenly by three, explains why Mother failed when she tried a moment ago to distribute her strawberries evenly among her children without cutting any.

Notice that in the latter explanation, the explanandum concerns Mother's failure in a particular attempt, whereas in the former explanation, the explanandum is more general.

Pincock ([2007]) offers another example. Why has no one ever succeeded (or: why did a given person on a given occasion not succeed) in crossing all of

the bridges of Königsberg exactly once (while remaining always on land or on a bridge rather than in a boat, for instance, and while crossing any bridge completely once having begun to cross it)? Here it is understood that the problem concerns the town's bridges as they were arranged when Euler considered this problem in 1735 (Figure 1). The distinctively mathematical explanation is that in the bridge arrangement, considered as a network, it is not the case that either every vertex or every vertex but two is touched by an even number of edges. (In fact, none is: one is touched by five edges, and each of the other three is touched by three edges.) Any successful bridge-crosser would have to enter a given vertex exactly as many times as she leaves it unless that vertex is the start or the end of her trip. So among the vertices, either none (if the trip starts and ends at the same vertex) or two could touch an odd number of edges. This explanation seems distinctively mathematical; the explanation would not have been distinctively mathematical if it had been that no one ever turned left rather than right after crossing a given bridge, or the bridges were made of a corrosive material, or someone was poised to shoot anyone who tried to cross a given bridge.

As another example, consider why a given attempt—or every past attempt, or every attempt ever—failed to unknot a trefoil knot (Figure 2) without cutting it? The distinctively mathematical explanation is that in three dimensions, the trefoil knot is distinct from the unknot. This explanation seems sharply different from an appeal to the fact that the knot was too tight, or that the rope was too hot to touch, or that all of those who tried gave up before
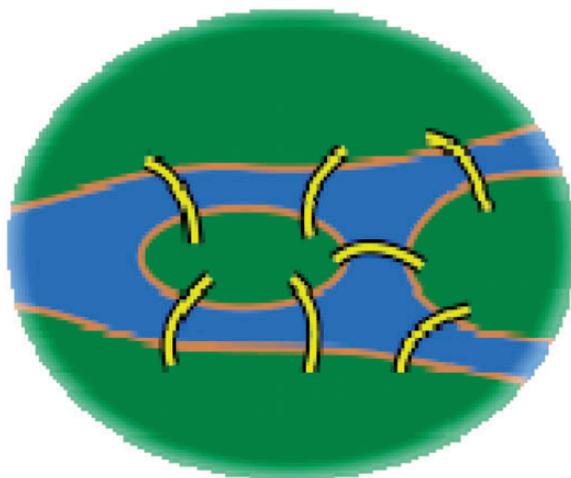
Figure 1. The bridges of Königsberg.
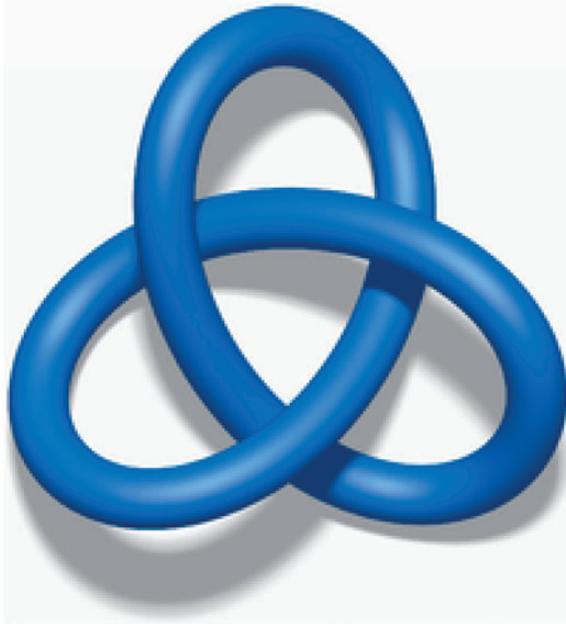
Figure 2. A trefoil knot.

they tried twisting the rope in a certain way—each of which might explain why every attempt to disentangle a certain knot failed.[2]

Lipton gives the following example:

> Suppose that a bunch of sticks are thrown into the air with a lot of spin so that they twirl and tumble as they fall. We freeze the scene as the sticks are in free fall and find that appreciably more of them are near the horizontal than near the vertical orientation. Why is this? The reason is that there are more ways for a stick to be near the horizontal than near the vertical. To see this, consider a single stick with a fixed midpoint position. There are many ways this stick could be horizontal (spin it around in the horizontal plane), but only two ways it could be vertical (up or down). This asymmetry remains for positions near horizontal and vertical, as you can see if you think about the full shell traced out by the stick as it takes all possible orientations. (Lipton [2004], pp. 9–10)

Mancosu ([2008], p. 134) says that the mathematical explanation of physical phenomena is 'well illustrated' by Lipton's example.

Perhaps these few examples suffice to suggest that, as Steiner ([1978], p. 18) says, 'one senses a striking difference' between distinctively mathematical

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2 Kitcher ([1989], p. 426) mentions a different knot case as an example of non-causal explanation.

explanations in science and ordinary scientific explanations that use mathematics. In the following sections, I examine various ways in which we might try to capture this difference as well as various approaches to understanding how these distinctively mathematical explanations work. Ultimately, I argue that these explanations explain not by describing the world's causal structure, but roughly by revealing that the explanandum is more necessary than ordinary causal laws are. The Königsberg bridges as so arranged were never crossed because they *cannot* be crossed. Mother's strawberries were not distributed evenly among her children because they *cannot* be. A trefoil knot was never untied because it *cannot* be. These necessities are stronger than causal necessity, setting distinctively mathematical explanations apart from ordinary scientific explanations. Distinctively mathematical explanations in science work by appealing to facts (including, but not always limited to, mathematical facts) that are modally stronger than ordinary causal laws—together with contingent conditions that are contextually understood to be constitutive of the arrangement or task at issue in the why question.

Although it has been suggested that distinctively mathematical explanations in science are 'non-causal', this idea requires careful elaboration (as we will see). Likewise, Mancosu ([2008], p. 135) tries to capture the distinction between distinctively mathematical and ordinary scientific explanations by saying that the former 'is explanation in natural science that is carried out by essential appeal to mathematical facts'.[3] But this criterion fails to exclude many ordinary scientific explanations. For example (following Purcell [1965], p. 28), to explain why the electric field strength at a distance r from a long, linear charge distribution with uniform charge density λ is equal (in Gaussian CGS units) to $2\lambda/r$, we can integrate the contributions to the field (given by Coulomb's law) from all segments of the line charge. When the integral is simplified, it becomes $(\lambda/r) \int_{-\pi/2}^{\pi/2} \cos\theta d\theta$. The explanation then makes essential appeal to the mathematical fact that $\int_{-\pi/2}^{\pi/2} \cos\theta d\theta = 2$. But intuitively it is not a distinctively mathematical explanation in science.

An account of the distinction between distinctively mathematical explanations in science and ordinary scientific explanations using mathematics should do justice to the conflicting ways we may find ourselves pulled in trying to classify a given explanation. For example, we may not entirely share Mancosu's confidence that Lipton's explanation (concerning the tossed sticks) is distinctively mathematical. Perhaps what is doing the explaining there is a propensity of the stick-tossing mechanism (that it is equally likely

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[3] Mancosu's view is echoed by Baker ([forthcoming]), who suggests that mathematical explanations in science are 'scientific explanations that appeal to mathematical facts'.

to produce a tossed stick in any initial orientation) together with a propensity of the surrounding air molecules (that they are equally likely to push a tossed stick in any way). (After all, if the tossed sticks were instead all spinning uniformly about axes that lie in the horizontal plane, they would be as likely at any moment to be vertical as horizontal, contrary to what we observe.) If the explanans in Lipton's example includes these propensities, then the explanation seems more like the explanation of a fair coin's behavior in terms of the propensities of the chance set-up than like the other examples I have given of distinctively mathematical explanations in science. An account of distinctively mathematical explanations should help us to understand this example.

Most of the literature I cite concerning distinctively mathematical explanations in science has been motivated largely by indispensability arguments for the existence of mathematical entities. The basic thought behind these arguments is that, if scientific theories must quantity over numbers, functions, sets, and other mathematical entities, then in being committed to these theories, we are committed (by Quine's criterion of ontological commitment) to the existence of these abstract entities just as we are committed to the existence of the concrete unobservable entities posited by these theories. Some philosophers believe that this argument is strengthened by the fact that some scientific explanations are distinctively mathematical. Other philosophers believe that the mathematical entities figuring in these explanations are not doing explanatory work of the same kind as concrete unobservables are posited as doing (or that Quine's account of ontological commitment fails to apply to abstract entities). In any case, this debate is irrelevant here. Philosophers engaged in this debate have generally paid relatively little attention to the questions I am pursuing: how do these 'distinctively mathematical' scientific explanations differ from ordinary scientific explanations that use mathematics and how do they succeed in explaining?

Of course, we might say that a scientific explanation qualifies as 'distinctively mathematical' exactly when it uses mathematics in the manner that is exploited by these indispensability arguments—that is, exactly when the explanation must quantify over mathematical entities. However, on this way of using the term 'distinctively mathematical', the ordinary scientific explanation of the fact that an infinite uniform line charge's electric field strength is inversely proportional to r counts as distinctively mathematical. It quantifies over mathematical entities: the explanans includes the fact that there exists a function in which r appears solely as $(1/r)$ and that solves the integral generated by summing the contributions to the field from all segments of the line charge. Therefore, this way of using 'distinctively mathematical' does not help to capture the intuitive difference between the explanations we just looked at and ordinary scientific explanations that use mathematics.

3 Are Distinctively Mathematical Explanations Set Apart by their Failure to Cite Causes?

Mancosu ([2008], p. 135) regards distinctively mathematical explanations in science as 'counterexamples to the causal theory of explanation'. Lipton ([2004], pp. 9–10) and Kitcher ([1989], p. 426) agree. This thought might suggest that distinctively mathematical explanations are set apart from ordinary scientific explanations by their failure to specify the explanandum's causes. However, there are two problems with this suggestion: (i) many ordinary scientific explanations fail to give the explanandum's causes, and (ii) at least some distinctively mathematical explanations do cite the explanandum's causes. Although I agree with Mancosu, Lipton, and Kitcher that distinctively mathematical explanations in science are non-causal explanations, we must be careful not to join Colyvan ([1998], pp. 324–5) in identifying an explanation as non-causal just when 'it makes no appeal to causally active entities'.

For example, an explanation that appeals to an omission or absence is typically causal even though, strictly speaking (according to many philosophers), an omission is not a cause since it is not even an event. For example, Brandon ([2006], p. 321) says that when we explain why a given body is moving uniformly by citing the absence of any forces on it, we give a causal explanation that cites no causes. Likewise, many philosophers (such as Prior et al. [1982]) regard dispositions as causally impotent to bring about their manifestations. Nevertheless, many philosophers (e.g. Jackson and Pettit [1992], p. 10) believe that a body's water-solubility, for example, causally explains why it dissolved when it was immersed in water. Likewise, when the possession of various categorical properties by various entities (together with some natural laws) explains why some entity possesses a given disposition, the explanation may be causal even though the disposition's categorical base does not cause the disposition. For instance, the categorical ground for a key's power to open a distant lock resides in the key's structure and the lock's structure, but the lock's structure is not a cause of the key's power (on pain of action at a distance on the cheap).

Thus, many explanations that fail to cite causes of the explanandum are nevertheless ordinary causal explanations and are not distinctively mathematical explanations. In order to highlight the difference between these kinds of explanations, I will adopt a broad conception of what makes an explanation 'causal': it explains by virtue of describing contextually-relevant features of the result's causal history or, more broadly, of the world's network of causal relations. For example, consider the explanation (in Section 2) of the derivative law concerning the strength of the electric field of a uniform infinite line charge. This explanation works by deducing the explanandum from Coulomb's law. This is a causal explanation even

though 'the explanation of a general law by deductive subsumption under theoretical principles is clearly not an explanation by causes' (Hempel [1965], p. 352) since laws are not causes. Although this explanation cites no causes, it works by describing the world's causal structure—in particular, the causes of the electric fields of uniform infinite line charges and the way that the contributions made by those causes combine. Similarly, Jackson and Petit ([1990]) say that when a body's water-solubility explains why it dissolved when immersed in water or when the squareness of a peg of side L explains its failure to fit through a hole of diameter L, the explanans explains not by identifying the particular causally efficacious properties possessed by the immersed body or the peg (having to do with various forces and their causes), but rather by specifying that the immersed body or the peg possesses some properties that ground certain dispositions but are otherwise left unspecified.

   Although an explanation that fails to identify causes may still be a causal explanation, not all explanations are causal. For example, that Samuel Clemens and Mark Twain are identical explains (non-causally) why they have the same height, birth dates, and so forth. For scientifically more prominent examples, consider some of the explanations in physics that appeal to symmetry principles. Energy conservation is explained by temporal symmetry: every law of nature follows from laws that are invariant under arbitrary time shift (i.e. under the transformation $t \rightarrow t \pm a$ for arbitrary temporal interval a). Likewise, the Lorentz transformations are explained by spatiotemporal symmetries supplemented by the principle of relativity (that there is a reference frame S such that for any frame S' in any allowed uniform motion relative to S, the laws in S and S' take the same form) and that the 'spacetime interval' $I = ([\Delta x]^2 + [\Delta y]^2 + [\Delta z]^2 - c^2[\Delta t]^2)^{\frac{1}{2}}$ between any two events is invariant (i.e. equal in S and in S').

   All of these are non-causal explanations. Spatiotemporal symmetries and the principle of relativity do not describe causal relations, even at a coarse-grained or abstract level. Rather, they impose constraints on the laws of nature; they require that the laws of nature take a certain form. Roughly speaking, spatiotemporal symmetries demand that every law treat all points in space, directions in space, and moments in time alike. Similarly, the fact that the spacetime interval is invariant is not a matter of the world's network of causal relations. Rather, it is a characteristic of the spacetime in which those causal relations obtain. Indeed, in using this principle to explain why the Lorentz transformations hold, Lee and Kalotas ([1975], p. 436) are careful to point out that 'c' in the principle is merely an arbitrary constant with the dimensions of velocity. In particular, they say, 'c' is not to be identified with the speed of light or of any other physical process. To do so would make the explanation causal—that is, would incorrectly depict the Lorentz

transformations as dependent upon the features of a particular causal process. In contrast, the symmetry principles, conservation laws, coordinate transformation laws, and so forth are prior to the particular sorts of causal relationships that happen to populate spacetime (see Lange [2011], [forthcoming]).

If (as I believe) Mancosu, Lipton, and Kitcher are correct in deeming distinctively mathematical explanations in science to be non-causal, then those explanations cannot work by describing the world's network of causal relations. How, then, do they work? Ultimately, I will suggest (in Section 5) that they work very much like symmetry principles do in explaining: by constraining what there could be.

I have argued that an explanation that fails to cite any causes may nevertheless qualify as causal if it explains by describing the world's network of causal relations. By the same token, some distinctively mathematical explanations, though non-causal, nevertheless happen to cite the explanandum's causes. Even so, they qualify as non-causal because they do not derive their explanatory power from their success in describing the world's network of causal relations specifically.

For instance, that Mother had three children and twenty-three strawberries were causes of her failure a moment ago when she tried to distribute her strawberries evenly among her children. That these were causes of her failure is the common verdict of many different accounts of causal relations. For instance, Lewis's counterfactual account says that C causes E exactly when there is a chain of stepwise 'influence' from C to E, where C 'influences' E exactly when 'there is a substantial range C_1, C_2, ... of different not-too-distant alterations of C (including the actual alteration of C) and there is a range E_1, E_2, ... of alterations of E, at least some of which differ, such that if C_1 had occurred, E_1 would have occurred, and if C_2 had occurred, E_2 would have occurred, and so on' (Lewis [2007], p. 476). Such a pattern of counterfactual dependence obtains in this example: if Mother had had twenty-four strawberries (or two children and twenty-two strawberries), for instance, then she would not have failed. Alternatively, a manipulability account of causal relations (such as Woodward [2003]) says roughly that C is a cause of E exactly when systematic changes in E can be brought about by suitable interventions on C. Clearly, manipulation of the numbers of strawberries or children would bring about corresponding changes in the outcome of Mother's attempt. Likewise, that there are three children and twenty-three strawberries raises the probability of the outcome from what it otherwise would be (in accordance with probabilistic accounts of causal relations), and there is a causal process of 'maternal strawberry distribution' connecting the outcome to the initial conditions (in accordance with accounts inspired by Salmon [1984]). Nevertheless, I maintain that this explanation is non-causal because it does not work by

describing the outcome's causes or, more broadly, the world's network of causal relations.

That a distinctively mathematical explanation happens to cite facts about the explanandum's causes does not mean that it works by virtue of describing the explanandum's causes. In the distinctively mathematical explanation, Mother's having three children helps to explain her failure to distribute the strawberries evenly not by virtue of being a cause of her failure, but rather by virtue of helping to make her success mathematically impossible. By the same token, that twenty-three cannot be divided evenly by three supplies information about the world's network of causal relations: it entails that there are no causal processes by which twenty-three things are distributed evenly (without being cut) into three groups. But in the distinctively mathematical explanation of Mother's failure, the fact that twenty-three cannot be divided evenly by three does not possess its power to explain by virtue of supplying this information about causal processes in particular. The distinctively mathematical explanation does not exploit what the world's causal structure is like as a matter of mathematical necessity. Rather, it exploits what the world is like as a matter of mathematical necessity: the fact that twenty-three things cannot mathematically possibly be divided evenly (while remaining uncut) into three groups explains why no collection of twenty-three things is ever so divided. The mathematical fact entails that even a pseudoprocess rather than a causal process (and even a world without causal processes) cannot involve such a division of twenty-three things. The mathematical fact supplies information about the world's network of causal relations (just as any fact does: that the cat is on the mat tells us that the world's network of causal relations includes no events caused by the cat's being off the mat). But its supplying information about the world's causal network per se is not responsible for its explanatory power in the distinctively mathematical scientific explanation. (In contrast, in a causal explanation, a fact's supplying information about the world's causal network per se is responsible for its explanatory significance.)

4 Distinctively Mathematical Explanations do not Exploit Causal Powers

I have just suggested that distinctively mathematical explanations in science are non-causal even though some of them give causes of their explananda. These explanations qualify as non-causal because they do not work by describing the world's network of causal relations; their explanatory power arises in some other way. Even if they happen to appeal to causes, they do not appeal to them as causes—they do not exploit their causal powers. In particular, I now suggest, any connection they may invoke between a cause and the

explanandum holds not by virtue of an ordinary contingent law of nature, but typically by mathematical necessity.[4] Thus, mathematics enters distinctively mathematical explanations in science.

It may be objected that although these explanations use mathematical facts, they also exploit the causal powers of any causes to which they appeal. In the Königsberg bridge case, for example, the arrangement of bridges and islands initially (i.e. when some attempt to cross them all begins) helps to cause their arrangements at later moments (while the attempt is underway), and this fact is crucial to the mathematical explanation. Likewise, in the example involving Mother's distribution of strawberries to her children, the numbers of children and strawberries initially (when Mother begins her attempt) are causes of the numbers later. That bridges are not brought into existence or caused to disappear by people travelling over other bridges, and that strawberries are not caused to replicate by being distributed, reflect the causal powers of various things and are matters of contingent natural law, not mathematical necessity. These facts underlie the distinctively mathematical explanations I have given.

I reply that those distinctively mathematical explanations do not exploit these causal powers. Rather, the fixity of the arrangement of bridges and islands, for example, is presupposed by the why question that the explanation answers: why did this attempt (or every attempt) to cross this particular arrangement of bridges—the bridges of Königsberg in 1735—end in failure? The bridges' arrangement does not function in connection with the distinctively mathematical explanation as an initial condition that happens to persist (partly by virtue of various causal laws) during all attempts to cross the bridges. Rather, the why question itself takes the arrangement as remaining unchanged over the course of any eligible attempt. If, during an attempt, one of the bridges collapsed before it had been crossed, then that journey would simply be disqualified from counting as having crossed the intended arrangement of bridges. The laws giving the conditions under which the bridges' arrangement would change thus do not figure in the explanans. (Likewise, it is understood in the why question's context that the relevant sort of 'crossing' involves a continuous path.)

If every distinctively mathematical explanation in science used no laws of nature, then this feature would nicely distinguish these explanations from many ordinary scientific explanations that use mathematics, such as the explanation (in Section 2) of any infinite uniform line charge's electric field strength. It would also distinguish distinctively mathematical

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[4] I presume the received view: the necessity possessed by ordinary laws of nature is weaker than mathematical necessity. Some philosophers argue that natural laws are metaphysically necessary and so are modally on a par with mathematical facts (and with the fact that Mark Twain and Samuel Clemens are identical). I examine this view in (Lange [2009a]).

explanations from many non-causal scientific explanations that are not distinctively mathematical, such as the explanations (in Section 3) of various conservation laws by symmetry principles (which are natural laws). However, it would not distinguish distinctively mathematical explanations from all non-causal explanations. For example, the explanation of the fact that Mark Twain and Samuel Clemens have the same height appeals to no natural laws. Furthermore, even if every distinctively mathematical explanation in science used no laws of nature, this feature would not distinguish distinctively mathematical explanations from all causal explanations; some causal explanations appeal to no natural laws. For example, a given biological trait's increasing frequency in some population may be explained by the absence of mutations and migrations together with that trait's fitness exceeding the fitness of any alternative to it that was initially represented in the population. Such an explanation appeals to the principle of natural selection (roughly speaking: fitter traits are more likely to spread, in the absence of mutation or migration). This principle is a broadly logical truth rather than a natural law (presuming all natural laws to be contingent), but it is not a mathematical fact.

That one trait is fitter than another entails that there is selection *of* the fitter trait but not that there is selection *for* that trait (Sober [1984]). That is, the fitter trait might not make various creatures more likely to have a greater number of viable offspring, but merely tend to be associated with traits that do. A selectionist explanation of a trait's increasing frequency (or of its current high frequency in the population) might go beyond the trait's greater fitness to identify the particular selection pressures at work; it might specify whether or not the given trait has been selected for and, if so, why. Baker ([2005], pp. 229–35, [2009]) characterizes one such selectionist explanation as a distinctively mathematical explanation in science. Although Batterman ([2010], p. 3) finds Baker's example 'interesting and persuasive' and Leng ([2005], p 174) agrees that it qualifies as a distinctively mathematical explanation of a physical phenomenon, I think we must first draw some distinctions before we can find here a distinctively mathematical explanation in science.

The explanandum in Baker's example is 'that cicada life-cycle periods are prime' rather than composite numbers of years (Baker [2009], p. 624). One possible explanation that biologists have offered is that a species with a periodic life-cycle maximizes its chance of avoiding predator species that also have periodic life-cycles exactly when the species' period in years is coprime to the most numbers close to it (where natural numbers m and n are 'coprime' exactly when they have no common factors except 1). That is because if two species' periods m and n are coprime, then their coincidence is minimized (since $m \times n$ is their lowest common multiple). Since a prime

number *m* is coprime to the most numbers close to it (namely, to every number less than 2*m*), it is evolutionarily advantageous for cicada life-cycle periods to be prime, and so (if this is the only relevant consideration) they are likely to be prime.

If this is the reason why cicada life-cycle periods are prime numbers of years, then this explanation works by describing the world's network of causal relations—in particular, the natural history of cicadas. Consider the explanation that cicadas have prime periods because prime periods have been selected for. This is a causal explanation, since 'selection *for* is the causal concept *par excellence*' (Sober [1984], p. 100). Likewise, consider the explanation that prime cicada periods are selected for over composite periods because some of the cicada's predators also have periodic life-cycles, the avoidance of predation by these predators is selectively advantageous to cicadas, and prime cicada periods tend to minimize this predation while bringing to cicadas no selective disadvantages that outweigh this advantage. This explanation is also just an ordinary causal explanation. It uses a bit of mathematics in describing the explanandum's causal history, but it derives its explanatory power in the same way as any other selectionist explanation. Taken as a whole, then, it is not a distinctively mathematical explanation.

But suppose we narrow the explanandum to the fact that in connection with predators having periodic life-cycles, cicadas with prime periods tend to suffer less from predation than cicadas with composite periods do. This fact has a distinctively mathematical explanation (as given above).[5] Analogous remarks apply to the selectionist explanation that Lyon and Colyvan ([2008], pp. 228–9) characterize as a distinctively mathematical explanation in science: 'What needs explaining here is why the honeycomb is always divided up into hexagons and not some other polygons (such as triangles or squares), or any combination of different (concave or convex) polygons' (Lyon and Colyvan [2008], p. 228). The explanation is that it is selectively advantageous for honeybees to minimize the wax they use to build their combs—together with the mathematical fact that a hexagonal

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[5]   Presumably, we would be prompted to ask for an explanation of this fact only as a result of having used this fact to help explain why cicada life-cycle periods are prime, an explanation that (I have just suggested) is causal. But it is not generally the case that the facts having distinctively mathematical explanations arise in science only by figuring in causal explanations. Some facts having distinctively mathematical explanations (such as the repeated, frustrating failure to untie trefoil knots) might be recognized and prompt a why question without any motivation from some broader causal explanatory project. Other facts having distinctively mathematical explanations (such as that there are always antipodal equatorial points at the same temperature) would presumably never have been noticed at all had they not been recognized as having distinctively mathematical explanations.

grid uses the least total perimeter in dividing a planar region into regions of equal area (the 'Honeycomb Conjecture' proved in 1999 by Thomas Hales). Again, this explanation works by describing the relevant features of the selection pressures that have historically been felt by honeybees, so it is an ordinary, causal explanation, not distinctively mathematical.[6] But suppose we narrow the explanandum to the fact that in any scheme to divide their combs into regions of equal area, honeybees would use at least the amount of wax they would use in dividing their combs into hexagons of equal area (assuming combs to be planar regions and the dividing walls to be of negligible thickness). This fact has a distinctively mathematical explanation: it is just an instance of the Honeycomb Conjecture. (By the same token, 'word problems' in mathematics textbooks are full of allusions to facts that are explained as immediate applications of mathematical facts, such as the fact that if Farmer Brown, with fifty feet of negligibly thin and infinitely bendable fencing, uses the fencing to enclose the maximum area in a flat field, then Brown arranges it in a circle.)

Of course, to allow Baker's original cicada explanation and Lyon and Colyvan's original honeybee explanation to qualify as 'distinctively mathematical', we could decide to use 'distinctively mathematical explanation' in a more inclusive sense to characterize any scientific explanation with a distinctively mathematical 'part'. However, many ordinary scientific explanations that use mathematics would then automatically qualify as 'distinctively mathematical'. For example, consider the standard explanation of the fact that a quantity of weightless, uniform, incompressible liquid, feeling only intermolecular forces, is in stable equilibrium exactly when it is spherical. This explanation ultimately appeals to the mathematical fact that for a given volume, the sphere is the shape that minimizes surface area. Is that enough to make the explanation qualify as distinctively mathematical, no matter what the rest of the explanation may be like?

5 How these Distinctively Mathematical Explanations Work

I suggested (in the previous section) that if every distinctively mathematical explanation in science used no laws of nature, then this feature would nicely

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[6] This explanation also seems to require that it be selectively advantageous for honeybees to divide their combs into regions of equal area. Lyon and Colyvan do not say why this might be. Moreover (as Alan Baker kindly pointed out to me), honeycomb cells in three dimensions are not quite the most economical shape, though they apparently come very close to optimality. Perhaps no honeybees ever actually possessed the neural architecture for producing the optimal shape, or perhaps that architecture was too costly in other ways and so it was overall most advantageous for honeybees to produce a cell shape that uses slightly more wax than necessary, or perhaps random drift is responsible for the outcome.
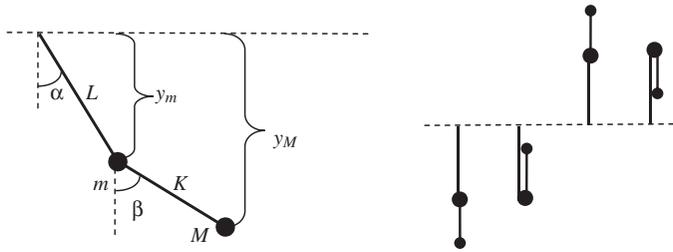
Figure 3. The simple double pendulum and its four equilibrium configurations (only the first is stable).

distinguish these explanations from many (though not all) ordinary scientific explanations that use mathematics, such as the explanation of an infinite uniform line charge's electric field strength. However, some distinctively mathematical explanations in science appeal to natural laws. Here is an example: Suppose we make a 'simple double pendulum' by suspending a simple pendulum from the bob of another simple pendulum and allowing both bobs to move under the influence of gravity (which varies negligibly with height) while confined to a single vertical plane (Figure 3). By definition, a 'simple pendulum' has an inextensible cord with negligible mass and encounters negligible friction and air resistance. The simple double pendulum has exactly four equilibrium configurations (Figure 3), where a 'configuration' is fixed by the angles α and β. (An 'equilibrium configuration' is a configuration where the two bobs, once placed there at rest, will remain there as long as the system is undisturbed.)

One way to explain why there are at least four equilibrium configurations is to identify the particular forces on the two bobs (with masses m and M as shown in Figure 3) and then to determine the configurations in which both bobs feel zero net force. By Newton's second law of motion, they will then undergo no acceleration and so will remain at rest once placed in that configuration.[7] Equivalently, since the force on a system is the negation of its potential energy's gradient,[8] we can express the system's potential energy $U(\alpha, \beta)$ and then solve for the configurations where the energy's gradient is

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[7] I presume throughout that classical physics is correct and so that this is a genuine explanation, that 'Newton's second law' is a genuine natural law, and so forth.

[8] That is, the force points in the direction in which the potential energy would diminish most steeply; the force's magnitude equals the rate of the potential energy's decrease in that direction (a partial derivative).
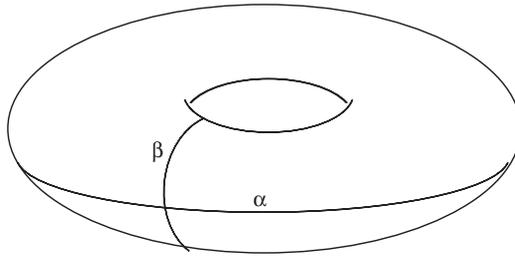
Figure 4. The topology of the configuration space of a simple double pendulum.

zero, i.e. where $\frac{\partial U}{\partial \alpha} = \frac{\partial U}{\partial \beta} = 0$—that is to say, where U is 'stationary' (i.e. at a maximum, minimum, or saddle point):

$$U(\alpha, \beta) = -mg\, y_m - Mg\, y_M$$
$$y_m = L\cos\alpha$$
$$y_M = L\cos\alpha + K\cos\beta$$
$$\text{So } U(\alpha, \beta) = -mgL\cos\alpha - Mg\,(L\cos\alpha + K\cos\beta)$$
$$\frac{\partial U}{\partial \alpha} = mgL\sin\alpha + MgL\sin\alpha$$
$$\frac{\partial U}{\partial \beta} = MgK\sin\beta$$

Hence, U's gradient vanishes exactly when $\sin \alpha = \sin \beta = 0$, which is exactly where $(\alpha, \beta) = (0,0)$, $(0,\pi)$, $(\pi,0)$, or (π,π)–the four equilibrium configurations shown in Figure 3.

This is a causal explanation.

There is also a non-causal, distinctively mathematical explanation. Since (α, β) and $(\alpha + 2\pi n, \beta + 2\pi m)$ designate the same configuration (for any integers n, m), the configuration space of any double pendulum can be represented as the points on a toroidal surface (Figure 4). Since $U(\alpha, \beta)$ is everywhere finite and continuous, it can be represented by distorting the torus so that each point (α, β)'s height equals $U(\alpha, \beta)$. Any such distortion remains a surface of genus $g = 1$ (i.e. topologically equivalent to a torus, which is a sphere with $g = 1$ holes in it), and for any such surface (smooth, compact, orientable, and so on), the numbers of minima, maxima, and saddle points obey the equation $N_{min} - N_{sad} + N_{max} = 2 - 2g$, which equals zero for $g = 1$.[9] By compactness, there must be at least one maximum

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[9] This equation for $g = 0$ (topologically equivalent to a sphere) is sometimes called the 'mountaineer's equation' and was proposed by Maxwell in 'On Hills and Dales' in 1870, expanding on work by Cayley. The general equation was proved by Morse in 1925 and derives from Möbius's 'The Theory of Elementary Relationships' in 1863.

and one minimum, so by this equation, there must be at least two saddle points and so at least four stationary points in total.

This is a non-causal explanation because it does not work by describing some aspect of the world's network of causal relations. No aspect of the particular forces operating on or within the system, which would make a difference to $U(\alpha, \beta)$, matters to this explanation. Rather, it exploits merely the fact that by virtue of its being a double pendulum, the system's configuration space is the surface of a torus—i.e. that U is a function of α and β. Consequently, the very same explanation applies to any double pendulum, not just to a simple one. For example, the same explanation applies to a compound square double pendulum (Figure 5), to a double pendulum where the two suspended extended masses are not uniformly dense, and to a complex double pendulum under the influence of various springs forcing its oscillation. Each of these has at least four equilibrium configurations, though the particular configurations (and their precise number) differ for different types of double pendulum. While the causal explanations differ for each of these kinds of pendulum (since the potential energy functions differ), the non-causal explanation is the same in each case.

Although this explanation does not work by describing the causes operating on the system, it does appeal to a natural law: a system is at equilibrium exactly when the net force on each of its parts is zero (i.e. when its potential energy is stationary)—a particular case of Newton's second law. Why doesn't this law make the explanation causal? Because, roughly speaking, Newton's second law describes merely the framework within which any force must act; it does not describe (even abstractly) the particular forces acting on a given situation. Any possible force accords with Newton's second law. For example, had gravity been an inverse-cube force, then it would still have operated according to Newton's second law. Had there been some other (physically impossible) kinds of forces in addition to the actual kinds, Newton's second
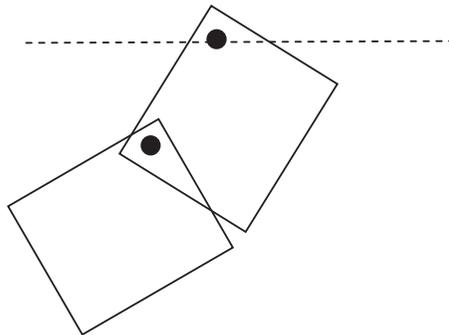
Figure 5. A compound square double pendulum.

law would still have held. Such counterlegals are sometimes invoked in science, as when Ehrenfest in 1917 famously showed that had gravity been an inverse-cube force or fallen off with distance at any greater rate, then planets would eventually have collided with the sun or escaped from the sun's gravity. Ehrenfest's argument requires that Newton's second law would still have held had gravity been an inverse-cube force or fallen off with distance at any greater rate. Newton's second law (like the conservation laws, the parallelogram law for the composition of forces, and the spacetime coordinate transformation laws) 'transcends' (Wigner [1972], p. 13) the peculiarities of the various kinds of forces there happen to be (e.g. electromagnetic, gravitational) in that it would still have held even if those forces had been different.

Indeed, to say 'the peculiarities of the various kinds of forces there happen to be' is to recognize that although these individual force laws are matters of natural necessity, Newton's second law is more necessary even than they. Compared with it, they happen to hold. In other words, although Newton's second law is not a mathematical, conceptual, metaphysical, or logical truth, it stands closer to them modally than an ordinary law does. Thus, an explanation that shows the explanandum to follow entirely from such laws thereby shows the explanandum to be necessary in a way that no explanation could do that depends on a force law (or, more broadly, in a way that no causal explanation could do).[10]

Any causal explanation in terms of forces (or energy) must go beyond Newton's second law to describe the particular forces at work (or energy function in play)—if not specifying them fully, then at least giving their relevant features (such as their proportionality to the inverse-square of the distance). That is why the distinctively mathematical double-pendulum

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[10] In (Lange [2009a]), I have elaborated more precisely what it would be for one stratum of laws to be more necessary than another, what these varieties of natural necessity have in common (and also share with mathematical necessity, for example) such that they are all species of the same genus, and why these varieties must come in a unique ordering by strength. In (Lange [2011]), I discuss the modal strength of the conservation laws in particular. My focus in (Lange [2009b]) is on the parallelogram of forces. In (Lange [forthcoming]), my argument centers on the spacetime transformations.

   As I explain in (Lange [2009a]), that one law *p* possesses a stronger variety of natural necessity than another law *q* is compatible with there being possible worlds where *q* holds but *p* does not. I explain there that the range of possible worlds where *p* holds *in connection with its necessity* nevertheless includes and extends beyond the range of possible worlds where *q* holds *in connection with its necessity*.

   Though my own account (Lange [2009a]) of natural law is non-Humean, even an advocate of a Humean account could presumably recognize certain laws as more necessary than others while giving some Humean account of what those varieties of necessity consist in. After all, Lewis ([1999], p. 232) has written: 'If you're prepared to grant that theorems of the best system are rightly called laws, presumably you'll also want to say that [...] they and their consequences are in some good sense necessary'. Accommodating several varieties of natural necessity, differing in strength, seemingly poses no new difficulties for a Humean beyond those involved in accommodating one variety of natural necessity.

explanation I have just described is non-causal despite including Newton's second law. The natural laws in a distinctively mathematical explanation in science, I suggest, must transcend the laws describing the particular kinds of causes there are. A distinctively mathematical explanation in science works not by describing the world's actual causal structure, but rather by showing how the explanandum arises from the framework that any possible causal structure must inhabit, where the 'possible' causal structures extend well beyond those that are logically consistent with all of the actual natural laws there happen to be.

An outcome having a distinctively mathematical explanation may also have a causal explanation. For example, the fact that a given double pendulum has at least four equilibrium configurations is explained both causally and non-causally. However, suppose we had two double pendulums: one simple, the other with inhomogeneous, extended masses and oscillations driven by various springs. Why do both of these pendulums have at least four equilibrium configurations? We could specify the energy functions for both pendulums and then derive separately the particular equilibrium configurations for each, thereby showing that each has at least four of them. But this derivation incorrectly portrays the explanandum as a coincidence (albeit a physically necessary one) since this derivation would fail to identify some important feature common to the two pendulums as responsible for their both having at least four equilibrium configurations. Since there is such a common feature, this derivation fails to explain why both pendulums have at least four equilibrium configurations. Only the distinctively mathematical explanation works; that these two double pendulums are alike in having at least four equilibrium configurations is no coincidence because any double pendulum must possess this property.[11] This must is stronger than the necessity possessed by the force laws, so the derivation from those laws cannot show that these two pendulums, just by virtue of being double pendulums, must have this feature.

In like manner, the distinctively mathematical explanation of the repeated failure to cross the Königsberg bridges shows that it cannot be done (where this impossibility is stronger than physical impossibility) and so that it was no

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[11] Kitcher makes a similar claim regarding the regularity (discovered by Arbuthnot) that in eighty-two successive years from 1623, more boys than girls were born in London. Suppose we took each birth during those years and described its causes (where one birth's causes have little in common with another's). By aggregating these, would we have thereby explained the regularity? 'Even if we had this 'explanation' to hand, and could assimilate the details, it would still not advance our understanding. For it would not show that Arbuthnot's regularity was anything more than a gigantic coincidence' (Kitcher [2003], p. 69), whereas in fact, it is no coincidence (as R.A. Fisher's evolutionary explanation reveals). However, I am not sure if Kitcher is claiming that the description of all of the causes is not an explanation of the regularity (as I believe, since it incorrectly depicts the regularity as coincidental) or merely that this causal 'explanation' is unilluminating.

coincidence that all actual attempts failed. The explanans consists not only of various mathematically necessary facts, but also (as we saw in Section 4) of various contingent facts presupposed by the why question that the explanandum answers, such as that the arrangement of bridges and islands is fixed. The distinctively mathematical explanation shows it to be necessary (in a way that no particular force law is) that, under these contingent conditions, the bridges are not crossed. By the same token, the distinctively mathematical explanation in the double pendulum example shows it to be necessary that a given double pendulum has at least four equilibrium configurations under certain contingent conditions (for example, that the string does not lengthen). These contingent facts specify the given double pendulum arrangement in question just as various contingent facts are understood to be presupposed by the task of crossing the Königsberg bridges.

Thus, I agree with Mancosu et al. (in the passages cited at the start of Section 3) that distinctively mathematical explanations in science are non-causal. But I do not accept Batterman's ([2010], p. 3) diagnosis that what makes these explanations non-causal is that they involve a 'systematic throwing away of various causal and physical details'. Although all of the distinctively mathematical explanations I have mentioned do abstract from such details, this feature does not make them non-causal. Many causal explanations do that, too—including explanations that appeal to one trait's having greater fitness than another (abstracting away from the detailed histories of individual mating, reproduction, and predation events), explanations that appeal to a peg's roundness and a hole's squareness (abstracting away from the particular intermolecular forces at work), and explanations (Jackson and Pettit [1992]) that appeal to a quart of boiling water cracking its flask (abstracting away from the particular collision between water and glass molecules that initiated the cracking). Likewise, although I agree with Pincock's characterization of the Königsberg bridge explanation as an 'abstract explanation' in that it 'appeals primarily to the formal relational features of a physical system' (Pincock [2007], p. 257), I do not agree with Pincock ([2007], p. 273) that 'abstract explanations' are a species of what are sometimes called 'structural explanations' (McMullin [1978]), since McMullin ([1978] p. 139) regards structural explanations as causal—as working by describing the constituent entities or processes (and their arrangement) that cause the feature being explained. In my view, the order of causal priority is not responsible for the order of explanatory priority in distinctively mathematical explanations in science. Rather, the facts doing the explaining are eligible to explain by virtue of being modally more necessary even than ordinary causal laws (as both mathematical facts and Newton's second law are) or being understood in the why question's context as constitutive of the physical task or arrangement at issue.

In elaborating how a distinctively mathematical explanation in science works, I have just tried to say why the role of Newton's second law in the double-pendulum explanation does not make the explanation causal. Now let's consider a slightly different question: why doesn't that role keep the explanation from qualifying as distinctively mathematical? Newton's second law is not a mathematical truth; it has greater necessity than an ordinary causal law (such as the gravitational-force law), but it is modally weaker than a mathematical fact. (All actual mathematical truths would still have held even if it had been the case that bodies of different materials but the same mass are affected in their motions in different ways by forces of the same magnitude and direction.) As we have seen, there are many non-causal scientific explanations. What makes some but not others qualify as distinctively mathematical? The answer cannot be simply that distinctively mathematical explanations appeal only to mathematical facts (along with contingent facts understood as specifying the physical task or arrangement at issue), not to natural laws, since the double-pendulum explanation is distinctively mathematical but appeals to Newton's second law.

I suggest that although a non-causal explanation appealing to *no* mathematical facts (such as the Twain-Clemens explanation) is not a distinctively mathematical explanation, there is no criterion that sharply distinguishes the distinctively mathematical explanations from among the non-causal explanations appealing to some mathematical facts. Rather, it is a matter of degree and of context. Insofar as mathematical facts alone are emphasized as doing the explaining, the explanation is properly characterized as distinctively mathematical. For example, the double-pendulum explanation is aptly characterized as distinctively mathematical in a context where the role of Newton's second law is not emphasized—that is, where the explanation is described as showing that the explanandum depends merely on the structure (specifically, the topology) of the system's configuration space (as in Chang et al. [1990], p. 179). The mathematical ingredient's prominence in the explanans is context-sensitive—unlike the explanation's character as non-causal.

Steiner ([1978]) offers a different proposal regarding the way in which distinctively mathematical explanations in science work. He suggests that such an explanation contains an argument that not only proves a given mathematical theorem, but also explains why that theorem holds—that is, constitutes a mathematical explanation *in mathematics*. This explanation is supplemented by a mapping from the mathematical theorem to a given physical case: the explanandum of the scientific explanation. In short, Steiner says, if all of the physics is removed from a mathematical explanation of a physical fact, then the remnant constitutes a mathematical explanation of a mathematical fact.

However, none of the mathematical explanations in science that I have mentioned incorporates a mathematical explanation in mathematics.

These mathematical explanations in science include mathematical facts, of course, but not their proofs—much less proofs that explain why those mathematical facts hold. For example, that three fails to divide twenty-three evenly explains Mother's failure to distribute twenty-three strawberries evenly among her three children, but this explanation includes no account of why three fails to divide twenty-three evenly. I am not sure that there even is a distinction between a proof that explains this fact and a proof that merely proves it. If ordinary, unremarkable mathematical facts like this one have no mathematical explanations, only proofs, then those facts would not thereby be prevented from funding distinctively mathematical explanations in science. Likewise, there are many mathematical proofs that the trefoil knot is topologically distinct from the unknot, but it is not evident to me which of them, if any, is explanatory. None of them is included in the distinctively mathematical explanation of the fact that no actual trefoil knot has ever been untied.[12]

An account of distinctively mathematical explanations aims to fit scientific practice by deeming to be explanatory only what would (if true) constitute genuine scientific explanations. Let's see what my account makes of the following non-explanation:

> Why are all planetary orbits elliptical (approximately)? Because each planetary orbit is (approximately) the locus of points for which the sum of the distances from two fixed points is a constant, and that locus is (as a matter of mathematical fact) an ellipse.

My account correctly deems this argument as failing to qualify as a distinctively mathematical explanation (presumably, it is no other kind of explanation either) because the first fact to which it appeals is neither modally more necessary than ordinary causal laws nor understood in the why question's context to be constitutive of being a planetary orbit (the physical arrangement in question).

How is it evident that a given feature is not so understood? In some cases, the features constitutive of the physical arrangement in question are alluded to in the why question (as in 'why has no one ever managed to cross *that* arrangement of bridges?'). But in other cases, they are not, and this may produce some ambiguity. As an example, let's return to Lipton's explanation concerning the tossed sticks (given in Section 2). We found ourselves uncertain as to whether it constitutes a distinctively mathematical explanation. It qualifies if the explanans does not include the 'evenness' of various propensities (for example, that the stick-tossing mechanism is equally likely to produce a tossed stick in any initial orientation). But if the explanans includes

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[12] Baker ([2009], pp. 623-4, [forthcoming]) makes the same point, though in terms of the cicada and honeycomb examples that I have argued do not straightforwardly qualify as distinctively mathematical explanations (and not in terms of my account of how distinctively mathematical explanations work).

these facts, then (we found) the explanation is just an ordinary causal explanation that works by describing various propensities. Which is it?

Lipton did not pose the why question in any particular context (other than that of a philosophy essay), and so the crucial part of this question was left unclear. Perhaps the 'evenness' of the tossing (i.e. the symmetry of the various probability distributions concerning the tossing mechanism and surrounding air) is tacitly part of the why question. In that case, the question asks why, in view of these symmetries, the outcome is so asymmetric (i.e. more sticks are nearly horizontal than nearly vertical). How does this asymmetry arise from such symmetry? The explanation is then distinctively mathematical. On the other hand, the why question may be requesting simply an appropriate description of the result's causes, without any emphasis on how the result's asymmetry contrasts with the setup's symmetry. The various propensities must then figure in the explanans. This ambiguity in Lipton's why question is the reason we find ourselves pulled in conflicting directions when we consider whether this explanation is distinctively mathematical.

6 Conclusion

I have tried to understand the kind of scientific explanation suggested by the examples given in Section 2—the kind that has figured in the literature on distinctively mathematical explanations in science. I have suggested that these explanations work not by describing the world's network of causal relations in particular, but rather by describing the framework inhabited by any possible causal relation. I have argued that this proposal not only captures the relevant intuitive distinctions and fits scientific practice, but also reveals what these distinctively mathematical explanations contribute to science that could not be supplied, even in principle, by causal explanations.[13]

Wesley Salmon famously contrasted the 'modal conception' of scientific explanation, according to which 'scientific explanations do their jobs by showing that what did happen had to happen' (Salmon [1985], p. 293), with the 'ontic conception' that he favored, 'according to which causality is intimately involved in explanation' (Salmon [1985], p. 296). He rejected the modal conception on the grounds that it is inapplicable to statistical explanation (Salmon [1985], pp. 295–6). But Salmon's objection shows only that some scientific explanations fall outside the modal conception, not that all do. I have argued that the modal conception, properly elaborated, applies at

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[13] Besides the kind of 'distinctively mathematical' explanation I discuss in this article, there are other kinds of scientific explanation that employ mathematics differently from the way that ordinary scientific explanations do. I examine one such explanation in (Lange [2010], pp. 332–7). It explains why two physically unrelated phenomenological laws are mathematically so analogous by revealing that they constitute essentially the same solution to the same mathematical problem.

least to distinctively mathematical explanation in science, whereas the ontic conception does not.

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